24.10.2021

Méthodes Numérique :

Soit a résoudre le système d'équations linéaires suivant :

$$\begin{cases} x + 3y + 5z = 10 \\ 2x + 5y + z = 8 \\ 2x + 3y + 8z = 3 \end{cases}$$

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 3 \end{pmatrix}$$

A b y

$$Ab = y$$

$$\underbrace{A' A}b = A' y$$

$$Ib = A'y$$

$$b = A'y$$

Script Python :

```
import numpy as np
A = np.array([[1,3,5],[2,5,1],[2,3,8]])
b = np.array([10,8,3])
Sol = np.linalg.Solve(A,b)
print(Sol)
```

→ pour résoudre le s[...] d'équations liné[...]

①

```
Ainv = np.linalg.inv(A)
Sol = Ainv @ b
print(Sol)
```

②

4 → $a(l, c)$

Résolution des systèmes particuliers :

$a(N, N)$

1/- Système à matrice diagonale :

Exp :

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix}$$

$$\begin{cases} 3x_0 + 0x_1 + 0x_2 = 9 \\ 0x_0 + 2x_1 + 0x_2 = 2 \\ 0x_0 + 0x_1 + 4x_2 = 2 \end{cases}$$

$$\begin{cases} x_0 = 9/3 = 3 \\ x_1 = 2/2 = 1 \\ x_2 = 2/4 = 1/2 \end{cases}$$

Cas général :

$$\begin{pmatrix} a_{00} & 0 & 0 & 0 \\ 0 & a_{11} & 0 & 0 \\ 0 & 0 & a_{ii} & \\ 0 & 0 & 0 & a_{nn} \end{pmatrix} \begin{pmatrix} x_0 \\ \\ x_i \\ x_n \end{pmatrix} = \begin{pmatrix} y_0 \\ \\ y_i \\ y_n \end{pmatrix}$$

Etapes :

1/ lire n (l'ordre de système)
2/ lire A (n,n)
3/ lire y(n)
4/ calculer x
5/ Afficher x

```
import numpy as np
n = int(input("Entrer l'ordre de système : "))
A = np.zeros((n,n))
for i in range(n):
    A[i,i] = float(input("A[" + str(i) + "i" + str(
y = np.zeros(n)
for i in range(n):  ← باش ترجعه 0 متجهة
    y[i] = float(input("y[" + str(i) + "]:"))
x = np.zeros(n)
for i in range(n):
    x[i] = y[i]/A[i,i]
print("La sol = ", x)
```

2) Système à matrice triangulaire inférieure :

Exp :
$$\begin{pmatrix} 3 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 4 \end{pmatrix}$$

$$\begin{cases} 3x_0 + 0x_1 + 0x_2 = 6 \\ 3x_0 + 2x_1 + 0x_2 = 8 \\ 1x_0 + 2x_1 + 2x_2 = 4 \end{cases}$$

$x_0 = 6/3 = 2 \rightarrow x_0 = 2$

$2x_1 = 8 - 3x_0 \rightarrow x_1 = \frac{8-6}{2} = 1 \rightarrow x_1 = 1$

$2x_2 = 4 - 1x_0 - 2x_1 \rightarrow x_2 = 0$

$$x = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a_{00} & 0 & 0 & \dots \\ a_{10} & a_{11} & 0 & \dots \\ a_{20} & a_{21} & a_{22} & \\ a_{i0} & a_{i1} & a_{i2} & \dots & a_{ii} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_i \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_i \end{pmatrix}$$

$x_i = ?$

$a_{00} x_0 = y_0$ .. 1

$a_{10} x_0 + a_{11} x_1 = y_1$ .. 2

$a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = y_2$ .. 3

$a_{30} x_0 + a_{31} x_1 + a_{32} x_3 = y_3$ .. 4

$\vdots$

$a_{i0} x_0 + a_{i1} x_1 + a_{i2} x_2 + \dots + a_{ii} x_i = y_i$

$a_{n0} x_0 + a_{n1} x_1 +$

① → $x_0 = y_0 / a_{00}$

② → $x_1 = (y_1 - a_{10} x_0) / a_{11}$

③ → $x_2 = (y_2 - a_{20} x_0 - a_{21} x_1) / a_{22}$

④ → $x_3 = (y_3 - a_{30} x_0 - a_{31} x_1 - a_{32} x_2) / a_{33}$

$x_i = (y_i - a_{i0} x_0 - a_{i1} x_1 - a_{i2} x_2 - \dots - a_{i\,i-1} x_{i-1}) / a_{ii}$

$x_n = (y_n - a_{n0} x_0 - a_{n1} x_1 - \dots - a_{n\,n-1} x_{n-1}) / a_{nn}$

$i = 0 \longrightarrow n$

$$x_i = \left(y_i - \sum_{k=0}^{i-1} a_{ik}\, x_k \right) / a_{ii}$$

Script pour mat triang inf :

```
# Ordre du sys
n = int(input("Entrer l'ordre du sys"))
# Lecture de A
A = np.zeros((n,n))
for i in range(n):
    for j in range(n): if (j <= i):
        A[i,j] = float(input("A[" + str(i)
# Lecture de y
y = np.zeros(n)
```

```
for i in range (n): y[i] = float(input ("y[" + str(i)+"]
# calcule de x
x = np.zeros (n)
for i in rang(n):
    S = 0
    for k in range (i)
    S = S + A [i, k] * x [k]
    x [i] = (y[i] - S) / A [i, i]
print (x)
```

3/. Système et matrice triangulaire supérieur

Exp :

$$\begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \\ 6 \end{pmatrix}$$

$$\begin{aligned} 2x_0 + 2x_1 + x_2 &= 4 \\ 2x_1 + 3x_2 &= 10 \\ 3x_2 &= 6 \end{aligned}$$

Par substitution arrière :

$x_2 = 6/3 \Rightarrow x_2 = 2$

$x_1 = \dfrac{10 - 3.2}{2} \Rightarrow x_1 = 2$

$x_0 = \dfrac{4 - 2x_1 - x_2}{2} \Rightarrow x_0 = \cancel{0}$ -1

$i = n \longrightarrow 0$

$$x_i = \left(y_i - \sum_{k=i+1}^{n} a_{ik}\, x_k \right) / a_{ii}$$

$$x_2 = y_2 / a_{22}$$

$$x_1 = (y_1 - a_{12}\, x_2) / a_{11}$$

$$x_0 = (y_0 - a_{01}\, x_1 - a_{02}\, x_2) / a_{00}$$

script mat triang sup:

```
for i in range (n-1, -1, -1):
    S = 0
    for k in range (i+1, n):
        S = S + A [i, k] * y [k]
    X [i] = (y [i] - S) / A [i, i]
```